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Sensitivity Analysis on Variation of Vane Natural Frequency of a Typical Aero Engine Impeller

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ABSTRACT
Centrifugal compressors are widely used in small and medium class turbo shaft and turbo prop aero engines. Inevitably the variation in the manufacturing process will lead to variation in natural frequency of blade vanes in an impeller which usually termed as Mistuning. The variation in natural frequency will increase the forced frequency response of the vanes than the tuned vanes. In-service deterioration of vanes dimension due to erosion, corrosion and FOD also adds to further variation in natural frequency. The amplification factor of the response will lead to reduced HCF life of the vanes than the tuned vanes. It is important for the designer to envisage the variation of natural frequency of the vanes to estimate the life of the impeller to avoid premature failure. This also helps in monitoring the health of the impeller during service.
This paper deals with prediction of the effects of manufacturing/geometry variation and variation in material properties on impeller vane natural frequency of a turbo shaft engine. FEM model is created to accommodate the geometry variation of the impeller. The parameters influencing the natural frequency are varied and its corresponding variation in frequency is predicted. Several conditions are also simulated and sensitivity analysis is carried out with the above result to predict the order of influencing parameters. Influencing parameters are ranked and the most influencing parameter is found to be the Young’s Modulus of the material for this case of impeller. Maximum influencing parameter Young’s modulus contributes to 5.8% variation and the least contributing factor density to 0.6% variation in natural frequency. This result enables the designer to forecast the possible range of natural frequencies in the design phase, so that he can limit the analysis to predict the response due to mistuning to the estimated range of frequencies.
Keywords – Impeller vane, Natural Frequency, Manufacturing Deviation, Sensitivity Analysis, High Cycle Fatigue

I. INTRODUCTION

Centrifugal compressors play a key role in aircraft propulsion, power generation and processes engineering application. Centrifugal compressors are considered to be robust, compact and combine these properties with comparatively high pressure ratios. In aerospace applications centrifugal compressors have been used for example in helicopter engines where high pressure ratios are needed to build compact and light-weight engines. Small and medium class turbo-prop and jet-engines have been relying on the integration of centrifugal stages to boost the pressure ratio downstream of an axial stage. As designers are striving towards higher efficiencies, mass flow rates and pressure ratios the centrifugal compressor design has evolved a lot. However, the continuous improvement in design towards aerodynamic attributes has been also pushing centrifugal compressor designs towards their structural limits. Aero engine safety is the most critical requirement as the design has to be robust to ensure safe operation and avoid mechanical failure. High cycle fatigue failure due to blade vibration which is faced by designers during the design of new products as well as during operation of existing machinery must be addressed in the design and development phase. HCF failure in engines is attributed by forced vibration of blades during engine operation. The response of the forced vibration may amplified because of Mistuning. Inevitably the variation in the manufacturing process will lead to variation in natural frequency of vanes in an impeller which usually termed as Mistuning. The variation in natural frequency will increase the forced frequency response of the vanes than the tuned vanes. In-service deteriotion of vanes dimension due to erosion, corrosion and FOD also adds to further variation in natural frequency. The amplification factor of the forced frequency response will lead to reduced high cycle fatigue life of the vanes than the tuned vanes. Since it affects the reliability of the engine, it is an important for the designer to envisage the variation of natural frequency of the vanes to estimate the life of the impeller to avoid premature and catastrophic failure. EL-Aini et al. [1] indicate that although 90% of the potential HCF problems are covered during development testing, the remaining few problems account for nearly 30% of the total development cost and are responsible for over 25%

of all engine distress events. Kielb [2] mentioned in 1998 that every new development program for jet engines has about 2.5 serious high cycle fatigue problems. Furthermore, Srinivasan [3] states in 1997 that the U.S. Air Force estimates an expenditure of about $100 million/year to inspect and fix high cycle fatigue related problems.

II. HIGH CYCLE FATIGUE

High Cycle Fatigue is one of the major causes of blade failures. There are two major loads acting on the blades, centrifugal load and vibration due to aerodynamic excitation that dictate the design of the impeller. Generally, the designers are working hard to maximise rotational speeds and on the other hand higher mass flow rates had to be realized. The latter has been facilitated through thinner and longer blades. However, the increase in rotational speed also increases the centrifugal load of the component leading to a rise in the static mean stress of the material. Since the thickness of the blade is becoming thinner compressor blades are vulnerable to vibration and it may suffer from high-cycle-fatigue (HCF) failure.

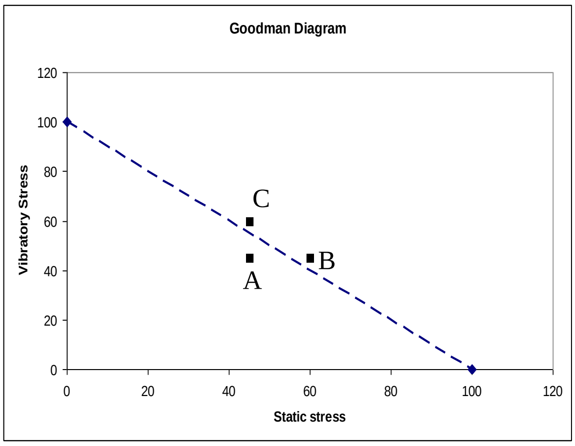

Figure-1: Goodman Diagram for Fatigue Analysis

Goodman diagram as shown in figure-1 is used to find the safe life of components for HCF life which defines an endurance limit under static and variable stresses. Static stresses arise from centrifugal force. Vibratory stresses are induced by the variation of aerodynamic forces while passing the nozzle. Depending on the equivalent stress of the two stress components may be within the endurance limit i.e. below the endurance line as indicated by A. With an increase in static stress as for example due to higher centrifugal loading, the static stress may move the overall stresses into a regime where the blade would fail as indicated by B. In a different scenario, unsteady blade excitation may increase i.e. due to changes in operating point and may thereby cause an increase in vibratory stress as indicated by C. In any event, operation outside the endurance limit represents a failure.

III. NATURAL FREQUENCY AND MODE SHAPE

Natural frequency is a property of a system frequency at which an object vibrates when excited by force. At this frequency, the amplitude of the vibration will amplify when excited by the external periodic force, which may lead to failure. Mode shape is shape of the vibrating structure at a given natural frequency. The eigenvalue (natural frequency) along with the eigenvector (mode shape) are calculated to understand the dynamics of a structure. A bladed disk or an impeller has many natural frequencies and associated mode shapes.

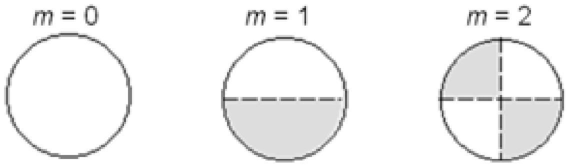

Figure-2: Schematic plot of Different Nodal Diameters

In the case of a bladed disk, the mode shapes have been described as nodal diameters (m). The term nodal diameter is derived from the appearance of a circular geometry, like a disk, vibrating in a certain mode.

Mode shapes contain lines of zero out-of-plane displacement which cross the entire disk as shown in figure-2. In other words, a node line is a line of zero displacement and the displacement is out of phase on the sides of the line represented by white and gray shades in figure-2. These are commonly called nodal diameters. Hence the natural frequency and nodal diameter are required to describe a bladed disk mode.

IV. BLADE VIBRATION AND SOURCES OF EXCITATION

Blade vibration in turbo machinery is an undesirable but also an inevitable mechanical phenomenon. It is of supreme importance with respect to restriction of component life. All rotating components within turbo machinery are subjected to vibration which is caused by periodic mechanical excitation or unsteady aerodynamic loading. The latter is of particular interest to blade vibration and has been subject to a number of investigations for the last decades. One aspect of research into this field was aimed towards understanding the excitation sources that cause blade vibration and their effect on the dynamic response of the blades. The other aspect was the necessity to estimate blade damping and identify damping contributions due to various damping mechanisms present in a blade assembly.

The maximum response of a vibrating structure during resonance that is subject to periodic excitation can be described by the following equation (a).

$$\eta_0 = \frac{|f|}{2w^2\zeta} \qquad \text{..............................(a)}$$

Where η_0 represents the response amplitude, f is the excitation amplitude; ζ is the critical damping ratio and w is the eigen frequency of the system. In order to calculate stresses from vibrating blade a accurate value of excitation forces as well as damping is required. Blade vibration due to forced response refers to excitation by external sources acting on the blade in the form of unsteady forces. Flutter refers to a form of self-excitation where a blade undergoes unsteady deformation and thereby introduces flow instabilities. Flutter is rarely encountered in centrifugal compressors as pointed out by Kushner [4] and Haupt [5].

A mathematical discussion of the condition of resonance is provided. In each revolution of a turbine wheel, blades pass through a field of pressure fluctuation due to nozzle or any other interruptions in the flow field. This fluctuation in pressure imposes a time varying force on the blades. In general such forces can be broken into harmonic components using Fourier analysis as in equation (b)

$$F = F_0 + F_1 \sin(w_1 t + \theta_1) + ... + F_n \sin(w_n t + \theta_n) \qquad \text{.... (b)}$$

The frequency of the harmonics depends on the speed of the turbine and the number of interruptions in the annulus like the number of nozzles, and is expressed as in equation (c)

$$w = \frac{NI}{60} \qquad \text{................................ (c)}$$

Where,
w = Driver frequency (HZ)
I = Number of nozzles around the wheel
N = Speed (RPM)

The frequency of any harmonic is an integer multiple of rotational speed and the number of interruptions. The nth harmonic of the force can be expressed as in equation (d):

$$f_n(\theta, t) = F_n \sin n(wt + \theta) \qquad \text{.............. (d)}$$

Where,
F_n = amplitude of the nth harmonic
θ = angular position on the disk
t = time

Resonance is achieved when the forces imposed on the blade do positive work. The work is defined as in equation (e)

$$W = \int_0^1 F dx \qquad \text{........................... (e)}$$

Where W is the work done by force F to move the body to a distance 1. The work done by the nth harmonic of the force on the mth nodal diameter of the bladed disk in one period can expressed as in equation (f)

$$W = \int_0^{2\pi}\int_0^{T} f_n(\theta, t) \frac{\partial}{\partial t} y_m(\theta, t) \frac{N}{2\pi} dt d\theta \qquad \text{......... (f)}$$

$$W = \begin{Bmatrix} \pi N F_n, \ for, n = m, w_m = w \\ 0, n \neq m, and, w_m \neq w \end{Bmatrix}$$

Where,
F_n = Amplitude of the nth Harmonic
y_m = Distance - constant
w_m = Natural frequency
θ = Angular position on disk
m = Nodal diameter
n = Harmonic index

The natural frequency w of the bladed disk must equal to the frequency of the driving force $w = w_m$ and the number of nodal diameters m must coincide with the harmonic of the force n i.e. $m = n$. The second result suggests that the work done will be zero when neither of the above conditions are satisfied which means no resonance will occur. This explains why the natural frequencies and driver frequency must match and also the force harmonic and structure mode shape or nodal diameter must match to achieve resonance.

V. MISTUNING

Mistuning is one of the major problems in blade vibration. The root of the problem is slight variations of blade-to-blade properties that cause eigen frequency shifts of the cyclic sectors. The effects of mistuning are encountered in cyclic symmetric structures i.e. bladed disk assemblies or impeller machined from a single piece of material. Mistuning affects the amplitude response of the vibrating blade. The amplification factor of the response is profound and might cause excess amplitudes that accelerates HCF failure. Lin [6] outlines the subject of wave propagation in periodic structures, initially he analyse ideal systems and then shows the dynamics of disordered structures, i.e. with slightly different properties. Periodic structures subject to mistuning suffer from localization, in which case essentially a large amount of the total energy is concentrated within a small region. In engineering practice this phenomenon becomes evident during testing and operation as failure of single blades. For this reason the subject of mistuning has received great attention in the past. Early work by Whitehead [7] pointed towards the fact that although the excitation may be regular, the mistuned system has multiple resonances with a distinct scatter in amplitude and phase. Reviews in great detail were given by Srinivasan [8] mentioned that small variation in individual frequencies from a datum frequency could result in intolerable levels of vibration. Mistuning is non-deterministic; therefore probabilistic methods have been initially adopted in order to assess the maximum response amplitude. Research into this field was largely driven by industry need to deal with this problem. Ideally "tuned" bladed disc maybe mistuned due to asymmetric friction of an assembly. Limitations have been associated with testing for the effect of mistuning, since bladed assemblies experience changes in properties during operation.

VI. MODAL ANALYSIS

To determine the natural frequencies of any structure and its corresponding mode shapes modal analysis is to be carried out. Cyclic symmetric impeller model exhibit certain well-defined types of vibration modes. These modes may be characterised by the number of equally spaced diametric nodal lines, n, typically referred to as *nodal diameters.* The maximum number of nodal diameters, which can occur, is $N/2$ if N is even and $(N-1)/2$ if N is odd, where N is the number of blades on the disk. This analysis is used to generate Campbell diagram and to verify the possible interference with engine orders. As we are studying the sensitivity of natural frequency of the impeller due to the possible manufacturing deviation, we carried out the modal analysis of the impeller for possibly deviated models. Apart from manufacturing deviation, abnormality formed during service due to erosion, corrosion and FOD are also considered for the analysis. Both addition and subtraction of mass of 5% and 10% from vanes are considered for the deviation in service. The analysis is also carried out for the model with deviation as per in service deviation. Totally seven vane models are made for analysis. Vane with nominal dimension, Maximum material condition, Minimum material condition, with thin root and thick tip, with thin tip and thick tip and 5% and 10% mass addition and subtraction to nominal blades are used for modal analysis. The variation in material properties are also considered for analysis.

Initially a single vane is modeled with nominal dimension for modal analysis to get the natural frequency and mode shapes without the effect of disc. This helps to separate out the blade natural frequency in the disc-vane coupled analysis. Modal analysis is carried out for single vane and the results are tabulated below.

Cyclic symmetry modal analysis is carried out on single sector bladed disc. The list of the natural frequencies corresponding to the nodal diameter (m) is shown in the table-2. The first mode frequency corresponds to the disc mode and mode 2 corresponds to the vane natural frequency.

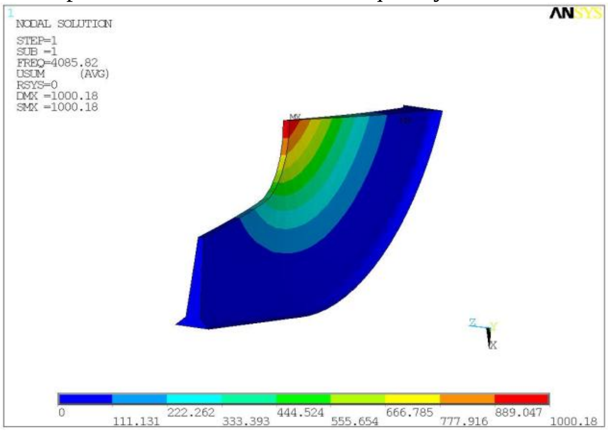

Figure-3: First Mode Shape of Single Impeller Vane

Mode	Frequency, Hz
1	4085
2	7540
3	9649
4	12144
5	13771

Table-1: Natural Frequency for five modes of single blade

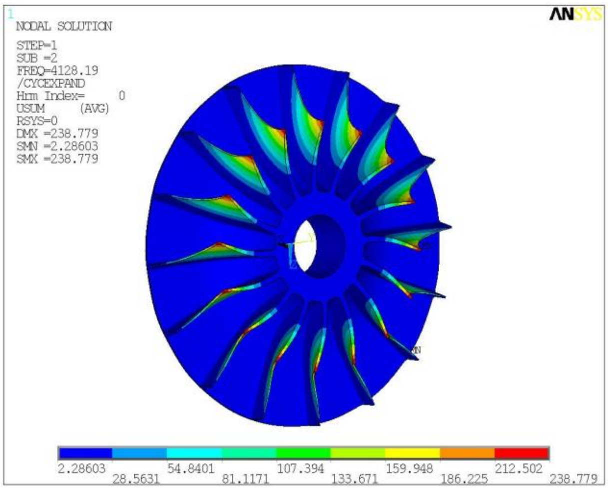

Figure-4: First Vane Mode Shape of Cyclic Symmetry Impeller

Mode	Natural Frequency, Hz				
	m=0	m=1	m=2	m=3	m=4
1	4090	3631	3576	3473	3290
2	4128	3938	4035	4125	4107
3	5157	5018	5103	5420	5792
4	7265	6483	6904	7027	7032
5	8199	7429	8366	8546	8687

Mode	Natural Frequency, Hz			
	m=5	m=6	m=7	m=8
1	3091	2915	2796	2753
2	4074	4048	4032	4027
3	6145	6422	6560	6592
4	7016	7051	7169	7246
5	8736	8678	8556	8487

Table-2: Natural Frequency of impeller with different

VII. HARMONIC ANALYSIS

In a structural system, any cyclic load will produce a harmonic response. This can be determined by Harmonic Analysis. Harmonic analysis results are used to determine the steady-state response of a linear structure to loads that vary sinusoidally with time, thus enabling us to identify the excitation frequency and find the response at the frequency which helps to overcome resonance, fatigue, and other harmful effects of forced vibrations. The impeller is subjected to harmonic analysis to find the first excitation frequency and its response. Variation of pressure on the vanes due to nozzle passing is taken as excitation force for harmonic response analysis. Cyclic symmetry model made with nominal dimension is used for the analysis and the response is shown in the figure-5.The response curve shows the most probable excitation frequency when the impeller undergoes a tap test.

VIII. SENSITIVITY ANALYSIS

Sensitivity analysis is a general concept which aim is to quantify the variations of an output parameter of a system with respect to changes imposed to some input parameters. The analysis is carried out by varying one parameter at a time to find the most influencing parameter. Modal analysis is carried out by varying individual parameters to its minimum and maximum range and the results are analysed to rank its sensitivity. The parameters which varied are density, Young's modulus and cases of geometric variations namely Max. material condition, Min. material condition, thick vane root and thin tip, thick vane tip and thin root. The frequency ranges are given below in the table-3. The result is used to rank the most contributing parameter for the natural frequency. From the analysis, it is noted that Young's modulus is the most influencing parameter for the variation of natural frequency in this impeller. The analysis results are shown in the rank of descending order in the figure-6.

Case	Parameter	Frequency Range, Hz
1	Young's Modulus	3982-4220
2	Density	4122-4146
3	Max. MC - Min. MC	4065-4120
4	Thick Tip-Thin Tip	4047-4148
5	Thick Root-Thin Root	4055-4105

Table-3: Variation of natural frequency with parameter

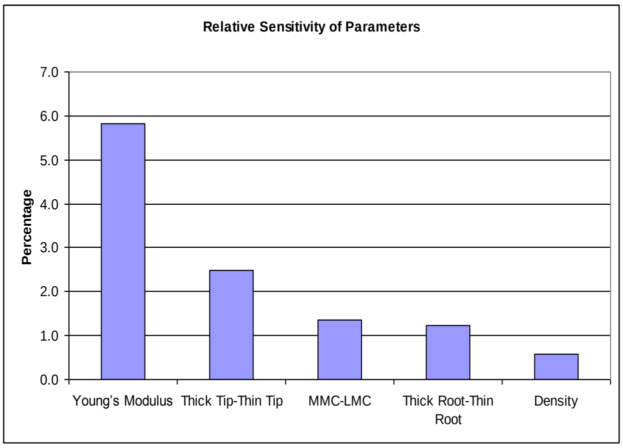

Figure-6: Sensitivity analysis result of the Impeller Vane

IX. CONCLUSION

The natural frequency of the impeller is determined by modal analysis. The parametric study on the impeller has been carried out to determine the most influencing parameter. It's found from the sensitivity analysis that Young's modulus is the most contributing factor for natural frequency ahead of the variation in impeller geometry variation. This study helps the designer to provide useful margin for the natural frequency and to avoid any interference during operation. The variation of natural frequency from the mean value is used to determine the amplification factor for initial mistuning analysis.

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